

Holographic and new agegraphic $f(T)$ -gravity models with power-law entropy correction

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Abstract

Using a correspondence between the $f(T)$ -gravity with the power-law entropy corrected versions of the holographic and new agegraphic dark energy models, we reconstruct the holographic and new agegraphic $f(T)$ -gravity models with power-law entropy correction. We also obtain the equation of state parameters of the selected models due to torsion contribution. Our results show that the equation of state parameters can accommodate the transition from the quintessence state to the phantom regime at recent stage.

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1 The $f(T)$ theory of gravity

The action of $f(T)$ modified teleparallel gravity is given by [1, 2]

$$I = \frac{1}{2k^2} \int d^4x e [f(T) + L_m], \quad (1)$$

where $k^2 = 8\pi G$ and $e = \det(e_\mu^i) = \sqrt{-g}$. Here e_μ^i is the vierbein field which uses as dynamical object in teleparallel gravity. Also T and L_m are the torsion scalar and the Lagrangian density of the matter inside the universe, respectively.

In the framework of $f(T)$ -gravity, the modified Friedmann equations in the spatially flat FRW universe are given by [3, 4, 5, 6, 7]

$$\frac{3}{k^2} H^2 = \rho_m + \rho_T, \quad (2)$$

$$\frac{1}{k^2} (2\dot{H} + 3H^2) = -(p_m + p_T), \quad (3)$$

where

$$\rho_T = \frac{1}{2k^2} (2T f_T - f - T), \quad (4)$$

$$p_T = -\frac{1}{2k^2} [-8\dot{H}T f_{TT} + (2T - 4\dot{H}) f_T - f + 4\dot{H} - T], \quad (5)$$

$$T = -6H^2, \quad (6)$$

and $H = \dot{a}/a$ denotes the Hubble parameter. Here ρ_m and p_m are the total energy density and pressure of the matter inside the universe, respectively. Also ρ_T and p_T are the energy density and pressure, respectively, due to the torsion contribution. The energy conservation laws are given by

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad (7)$$

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0. \quad (8)$$

For the special case $f(T) = T$, from Eqs. (4) and (5) $\rho_T = p_T = 0$ and the usual Friedmann equations in general relativity (GR) are recovered.

In the presence of the torsion contribution, the equation of state (EoS) parameter is defined as [7]

$$\omega_T = \frac{p_T}{\rho_T} = -1 + \frac{8\dot{H}T f_{TT} + 4\dot{H} f_T - 4\dot{H}}{2T f_T - f - T}. \quad (9)$$

In the subsequent sections, we reconstruct different $f(T)$ -gravities from the power-law entropy corrected holographic dark energy (PLECHDE) and new agegraphic dark energy (PLECNADE) models.

2 Power-law entropy corrected holographic $f(T)$ -gravity model

Here like [7, 8] we reconstruct the $f(T)$ -gravity according to the PLECHDE model. Following [9, 10] the PLECHDE density with the IR cut-off $L = R_h$ is given by

$$\rho_\Lambda = \frac{3c^2}{k^2 R_h^2} - \frac{\beta}{k^2 R_h^\alpha}, \quad (10)$$

where c , α and β are constants. For $\beta = 0$ the above equation transforms to the well-known HDE density [11, 12]. Also the future event horizon R_h is defined as

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}. \quad (11)$$

Following [13, 14] we assume two ansatzs for the scale factor. The first ansatz is given by

$$a(t) = a_0(t_s - t)^{-h}, \quad t \leq t_s, \quad h > 0. \quad (12)$$

Using Eqs. (6) and (12) one can get

$$H = \frac{h}{t_s - t}, \quad T = -\frac{6h^2}{(t_s - t)^2}, \quad \dot{H} = -\frac{T}{6h}. \quad (13)$$

For the second ansatz as

$$a(t) = a_0 t^h, \quad h > 0, \quad (14)$$

one can obtain

$$H = \frac{h}{t}, \quad T = -\frac{6h^2}{t^2}, \quad \dot{H} = \frac{T}{6h}. \quad (15)$$

For the scale factor (12) and using Eq. (13), the future event horizon R_h yields

$$R_h = a \int_t^{t_s} \frac{dt}{a} = \frac{t_s - t}{h + 1} = \frac{h}{h + 1} \left(\frac{-6}{T} \right)^{1/2}. \quad (16)$$

Replacing Eq. (16) into (10) yields

$$\rho_\Lambda = -\frac{\gamma}{2k^2}T - \frac{\delta}{2k^2}(-T)^{\frac{\alpha}{2}}, \quad (17)$$

where

$$\gamma = c^2 \left(\frac{h+1}{h} \right)^2, \quad (18)$$

$$\delta = 2\beta \left(\frac{h+1}{\sqrt{6}h} \right)^\alpha. \quad (19)$$

Equating (4) with (17), i.e. $\rho_T = \rho_\Lambda$, gives the following differential equation

$$2Tf_T - f - (1 - \gamma)T + \delta(-T)^{\frac{\alpha}{2}} = 0. \quad (20)$$

Solving Eq. (20) yields the power-law entropy corrected holographic $f(T)$ -gravity model as

$$f(T) = \epsilon T^{1/2} + (1 - \gamma)T + \frac{\delta}{1 - \alpha}(-T)^{\frac{\alpha}{2}}, \quad (21)$$

where ϵ is an integration constant.

Inserting Eq. (21) into (9) gives the EoS parameter of the torsion contribution as

$$\omega_T = -1 - \frac{1}{3h} \left(\frac{\alpha\delta(-T)^{\frac{\alpha}{2}}}{\gamma T + \delta(-T)^{\frac{\alpha}{2}}} \right), \quad h > 0. \quad (22)$$

The above relation shows that the EoS parameter is time-dependent. Hence it can justify the transition from quintessence state, $\omega_T > -1$, to the phantom regime, $\omega_T < -1$, as indicated by recent observations [15].

For the scale factor (14), using Eq. (15) the future event horizon R_h reduces to

$$R_h = a \int_t^\infty \frac{dt}{a} = \frac{t}{h-1} = \frac{h}{h-1} \left(\frac{-6}{T} \right)^{1/2}, \quad h > 1, \quad (23)$$

where the condition $h > 1$ is necessary due to having a positive future event horizon. For the scale factor (14), the resulting $f(T)$ is the same as (21) where

$$\gamma = c^2 \left(\frac{h-1}{h} \right)^2, \quad (24)$$

$$\delta = 2\beta \left(\frac{h-1}{\sqrt{6}h} \right)^\alpha. \quad (25)$$

Also the EoS parameter is obtained as

$$\omega_T = -1 + \frac{1}{3h} \left(\frac{\alpha\delta(-T)^{\frac{\alpha}{2}}}{\gamma T + \delta(-T)^{\frac{\alpha}{2}}} \right), \quad h > 1, \quad (26)$$

which is a dynamical EoS parameter. Hence it can also accommodate the transition from $\omega_T > -1$ to $\omega_T < -1$ at recent stage.

3 Power-Law entropy corrected new agegraphic $f(T)$ -gravity model

The PLECNADe density is given by [9]

$$\rho_\Lambda = \frac{3n^2}{k^2\eta^2} - \frac{\beta}{k^2\eta^\alpha}, \quad (27)$$

where n , α and β are constants. Also η is the conformal time of the FRW universe defined as

$$\eta = \int \frac{dt}{a} = \int \frac{da}{Ha^2}. \quad (28)$$

In the absence of correction term ($\beta = 0$), Eq. (27) reduces to the ordinary NADE density [16, 17].

For the scale factor (12), using (13), the conformal time η yields

$$\eta = \int_t^{t_s} \frac{dt}{a} = \frac{(t_s - t)^{h+1}}{a_0(h+1)} = \frac{h^{h+1}}{a_0(h+1)} \left(\frac{-6}{T} \right)^{\frac{h+1}{2}}. \quad (29)$$

Substituting Eq. (29) into (27) gives

$$\rho_\Lambda = \frac{\lambda}{k^2} (-T)^{h+1} - \frac{\sigma}{k^2} (-T)^{\alpha(\frac{h+1}{2})}, \quad (30)$$

where

$$\lambda = \frac{3n^2 a_0^2 (h+1)^2}{(6h^2)^{h+1}}, \quad (31)$$

$$\sigma = \beta \left(\frac{a_0(h+1)}{(\sqrt{6}h)^{h+1}} \right)^\alpha. \quad (32)$$

Equating (4) with (30) yields

$$2Tf_T - f - T - 2\lambda(-T)^{h+1} + 2\sigma(-T)^{\frac{\alpha}{2}(h+1)} = 0. \quad (33)$$

Solving Eq. (33) reduces to the power-law entropy corrected new agegraphic $f(T)$ -gravity model as

$$f(T) = \epsilon T^{1/2} + T + \frac{2\lambda}{1+2h}(-T)^{h+1} + \frac{2\sigma}{1-\alpha(1+h)}(-T)^{\frac{\alpha}{2}(h+1)}, \quad (34)$$

where ϵ is an integration constant. Replacing Eq. (34) into (9) gives

$$\omega_T = -1 - \frac{1+h}{3h} \left(\frac{-2\lambda + \alpha\sigma(-T)^{(\frac{\alpha}{2}-1)(h+1)}}{\lambda + \sigma(-T)^{(\frac{\alpha}{2}-1)(h+1)}} \right), \quad h > 0, \quad (35)$$

which is time-dependent and can justify the transition from $\omega_T > -1$ to $\omega_T < -1$.

For the scale factor (14), using (15) the conformal time η is obtained as

$$\eta = \int_0^t \frac{dt}{a} = \frac{t^{1-h}}{a_0(1-h)} = \frac{h^{1-h}}{a_0(1-h)} \left(\frac{-6}{T} \right)^{\frac{1-h}{2}}, \quad 0 < h < 1, \quad (36)$$

where the condition $h < 1$ is necessary due to having a positive conformal time. The resulting $f(T)$ is

$$f(T) = \epsilon T^{1/2} + T + \frac{2\lambda}{1-2h}(-T)^{-h+1} + \frac{2\sigma}{1-\alpha(1-h)}(-T)^{\frac{\alpha}{2}(1-h)}, \quad (37)$$

where

$$\lambda = \frac{3n^2 a_0^2 (1-h)^2}{(6h^2)^{1-h}}, \quad (38)$$

$$\sigma = \beta \left(\frac{a_0(1-h)}{(\sqrt{6}h)^{1-h}} \right)^\alpha. \quad (39)$$

Also the EoS parameter is obtained as

$$\omega_T = -1 - \frac{h-1}{3h} \left(\frac{-2\lambda + \alpha\sigma(-T)^{(\frac{\alpha}{2}-1)(1-h)}}{\lambda + \sigma(-T)^{(\frac{\alpha}{2}-1)(1-h)}} \right), \quad 0 < h < 1, \quad (40)$$

which can accommodate the transition from $\omega_T > -1$ to $\omega_T < -1$.

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